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SED FOR OPTIMAL ACQUISITION DESIGN AND SENSOR-TO-SAMPLE DISTANCE APPLIED TO SCANNING MAGNETIC MICROSCOPY

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ABSTRACT

Over the last decades, scanning magnetic microscopy techniques have been increasingly used in paleomagnetism and rock magnetism. Different from standard paleomagnetic magnetometers, scanning magnetic microscopes produce high-resolution maps of the vertical component of the magnetic induction field on a plane located over the sample. These high-resolution magnetic maps can be used to estimate the magnetization distribution within a rock sample by inversion. Previous studies have estimated the magnetization distribution within rock samples by inverting the magnetic data measured on a single plane above the sample. A recent work presented a spatial domain method for inverting the magnetic induction measured on four planes around the sample in order to retrieve its internal magnetization distribution. This methodology presumes that the internal magnetization distribution of the sample varies along one of its axes. Moreover, the sample geometry can be approximated by an interpretation model composed of a one-dimensional array of juxtaposed rectangular prisms with uniform magnetization. The Cartesian components of the magnetization vector within each rectangular prism are the parameters to be estimated by solving a linear inverse problem. This method takes an advantage on dealing with the averaged magnetic field due to the finite size of the magnetic sensor, preventing the application of a deconvolution before the inversion. Tests with synthetic data show the advantage of inverting the magnetic data on four planes around the sample and how this new acquisition scheme improves the estimated magnetization distribution within the rock sample. However, we have to analyze the influence of the scanning geometry and the number of measurements on each observation plane. In this work, we propose some strategies on determining optimal acquisition design and sensor-to-sample distance in order to improve practical use of this method based on Statistical Experimental Design. We applied this technique for constructing a Quality Factor map that defines previously an experimental procedure to set the two main acquisition parameters. Tests with synthetic data simulating a ferromanganese crust show the robustness of this technique on providing a supplementary tool to optimize the acquisition design of the method.

Keywords: Inverse problem; magnetic microscopy; paleomagnetism; statistical experimental design (SED).

1. Introduction

Based on estimating the total remanent magnetization, standard techniques in paleomagnetism and rock magnetism (Collinson, 1983) have been used. Usual magnetometers have become standard in state-of-art paleomagnetic laboratories. Although, these instruments have some limitations in the spatial variability of magnetization and low magnetized samples. The evolution of these studies depends on development of new instruments in order to improve the quality of measurements and processing data, as well as new techniques to estimate magnetization distribution of rock samples. In this context, a new generation of equipment is emerging to solve such limitations. Scanning Magnetic Microscopes (SMM) map the magnetic field on a plane over a rock sample, in general, measuring the vertical component of the field. These instruments have the greatest sensibility and spatial resolution for paleomagnetic studies (Weiss *et al.*, 2007; Egli and Heller, 2000).

Standard paleomagnetic techniques separate the original sample mechanically into a set of subsamples and estimate the magnetization of all slices (Butler, 1998). However, SMM devices allow a nondestructive characterization of internal magnetization distribution of rock sample by inverting high-resolution magnetic data. Despite this, the inverse problem aiming at estimating the magnetization distribution within rock sample require efficient matrix algorithms. Moreover, SMM can produce a huge set of data during the scanning stage. It is also well known that inverse problems are generally no unique, owing to infinite number of solutions that produce the same set of observed data. This ill-conditioning problem can be narrowed by introducing a priori information regarding the magnetization distribution and/or optimizing the geometry of the data acquisition. The introduction of a priori information aiming at constraining the possible estimated magnetization and also making them stable is called regularization (Aster *et al.*, 2005). Some recent works estimate the 2D magnetization distribution within planar rock samples (thin sections) by inverting the magnetic data measured on a plane above it. Egli and Heller (2000) propose a method, in the wavenumber domain, aiming at retrieving the magnetization within a planar sample. Lima *et al.* (2013) presented a method in the Fourier domain attempted to estimate the magnetization distribution defining a unidirectional magnetization of a planar rock sample. Reis *et al.* (2016) propose a spatial domain method to retrieve the tridimensional magnetization distribution within a rectangular rock sample formulating a linear inverse problem measuring the magnetic field on four planes around the sample.

In recent works, Statistical Experimental Design (SED) technique has been used to find optimal designs of acquisition. An optimal acquisition design contribute for improving the quality of a set of estimates, as well as by justifying the experimental cost in terms of robustness, accuracy and precision of geological interpretation. In other words, the design problem consists of finding how information about the model is maximized. That is, the acquisition design dictates the quality of information provided by practical implementations. For this reason, It is therefore critical to design experimental procedures in order to determine an effective cost of the experiment. Considering a linear inverse problem, a method to investigate an experimental design is using the Singular Value Decomposition of the sensitivity matrix. The choice of method to design experiments depends strongly on how easily one can measure information (Curtis, 2004). A set of Singular Value relates directly on how pieces of information can be estimated from the data to be collected. Curtis (2004) propose a set of Quality Factor (QF) based on a normalization of Singular Value spectrum to get some information about the survey used in a simple tomography design formulating a linear inverse problem.

Two essentials parameters of SMM survey are the sensor-to-sample distance and the spatial resolution used at the moment of scanning stage (*i.e.*, the number of observations over the observation plane). However, the method proposed by Reis *et al.* (2016) can be cost expensive due to the huge data set measured on four planes around the rock sample. For this reason, it is necessary to define an optimal combination between sensor-to-sample distance and spatial resolution to investigate the design acquisition previously. In this work, we use SED technique in order to improve practical use of Reis *et al.* (2016) method. In addition, we construct a Quality Factor map to produce a set of optimal parameters that can be previously chosen by the interpreter, to guide the experimental procedure. Furthermore, once the geometry of the sample and the dimensions of a squared magnetic sensor are defined, a set of optimal parameters can be nested from a quality factor map. A Synthetic test simulating a ferromanganese crust based on Oda *et al.* (2011) shows the performance on the application of SED technique in SMM.

2. Methodology

Reis *et al.* (2016) aproximate a rectangular rock sample to an interpretation model composed by juxtaposed rectangular prisms. The three Cartesian components of magnetization vector within each prism are the parameters to be estimated by solving a linear system given by

$$m^{est} = (M^T M)^{-1} M^T d^o \qquad [1]$$

Where **M** is the sensitivity matrix and $\mathbf{d^o}$ is the observed data. More details about the inverse problem formulation are presented in Reis *et al.* (2016). The sensitivity matrix depends on two main factors that will be analyzed in this work: the sensor-to-sample distance **h** and the spatial resolution **dx**. We use the Statistical Experimental Design (SED) technique to investigate the influence of these two factors on the acquisition design. We applied a linear SED method that is based on the maximization of the following quality measure proposed by Curtis (2004):

$$\Theta = \frac{\lambda_i}{\lambda_{max}} \qquad [2]$$

Where λ_i represent the i-th Singular Value and λ_{max} the maximum Singular value of the matrix $\mathbf{M^T M}$ in Eq. 1. This quality factor reflects the amount of information, that is expected to be transferred, from measured magnetic data to the parameters (Cartesian components of the magnetization distribution). Here, we use this quality measure to predict how the sensor-to-sample distance and the spatial resolution can affect the estimated magnetic distribution obtained with the method proposed by Reis *et al.* (2016). We also use the Quality Factor to construct a map to investigate which combination of sensor-to-sample distance and spatial resolution generate the maximum value of the Quality Factor. To construct this map, we calculate the QF on a regular grid using **Nh** number of sensor-to-sample distance and **Ndx** number of spatial resolution.

3. Results

We have simulated, based on Oda *et. al.* (2011), a synthetic sample formed by P = 24 juxtaposed prisms along the x-axis (Fig. 1). This sample has side lengths equal to Lx = 36 mm, Ly = 5 mm and Lz = 5 mm along x, y and z-axes, respectively. We have also simulated the presence of 20 highly magnetized with 100 A/m of magnetization intensity grains that are randomly placed within the sample. This synthetic sample also has a spatial magnetization distribution showed in Figure 4 (red dots). We also have simulated a planar squared magnetic sensor of dimension 300 μm.

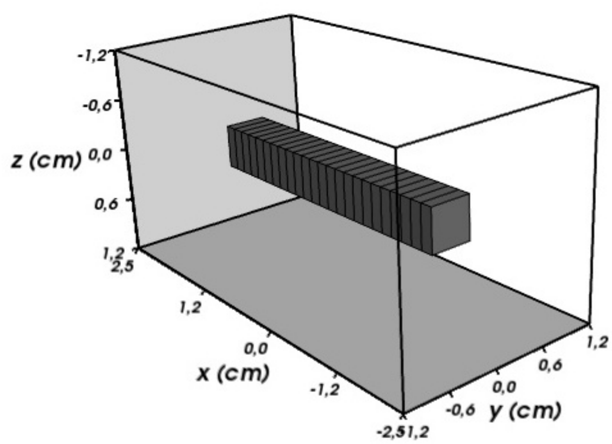

Figure 1. Simulation of ferro-manganese crust sample based on Oda *et al.* (2011).

3.1 Quality factor map, Singular Value Spectrum and the estimates

This section shows the Quality Factor Map using different sensor-to-sample distance and spatial resolution using four observation planes around the sample and illustrates the application of the SED technique for estimating magnetization distribution within rock samples. We compare the Singular Value Spectrum of different points on the QF map as well as the final result for the estimates.

We generate a regular grid using **Nh** = 10 and **Ndx** = 10 to construct the Quality Factor map (Fig. 2) varying the sensor-to-sample distance from 80 μm to 1000 μm and the spatial resolution from 100 μm to 1000 μm. We set two points on QF map on the maximum region with the values **h** = 300 μm/**dx** = 400 μm and the minimum region with the values **h** = 100 μm/**dx** = 800 μm. From each point, we calculate the Singular Value Spectrum (Fig. 3) and compare the result. We applied the method proposed by Reis *et al* (2016) for these two points and the results are presented in Figure 4 (blue dots and green dots).

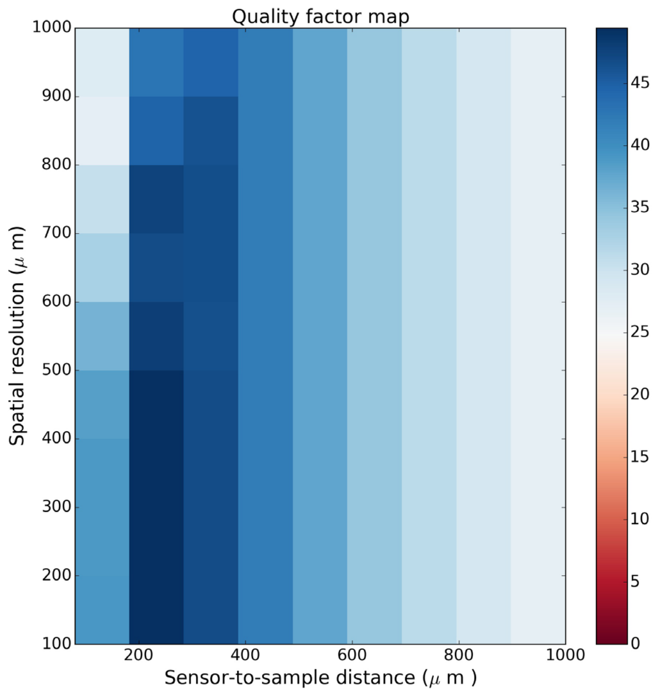

Figure 2. Quality Factor Map using four observation planes. Number of heights **Nh** = 10 along the x-axis (sensor-to-sample distance in microns) and number of spatial resolution **Ndx** = 10 along the y-axis (spatial resolution in microns). The range of height varies from 80 μm to 1000 μm. The range of spatial resolution varies from 100 μm to 1000 μm. The dark blue region shows a maximum value of the Quality Factor.

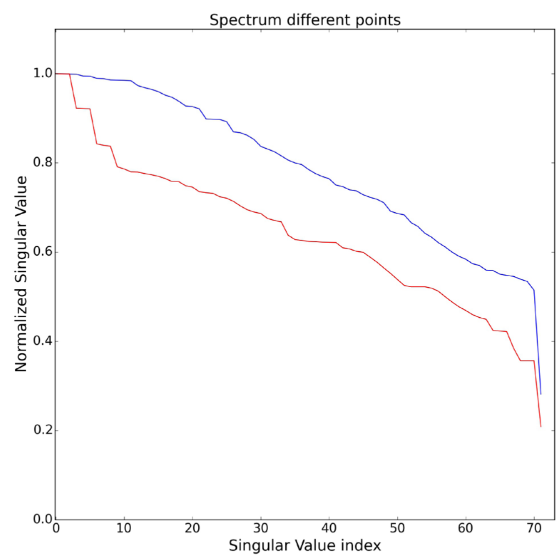

Figure 3. The Singular Value Spectrum for two points set on figure 2. Blue curve represents the maximum Quality Factor value in **h** = 300 μm/**dx** = 400 μm. Red curve shows a SV spectrum of a low QF value in **h** = 100 μm/ **dx** = 800 μm.

We analyze each result presented on Figures 2, 3 and 4. As shown in Figure 2, quality measure decrease when the sensor-to-sample distance increase and, on the other hand, a slight change on QF when spatial resolution decrease. Figure 3 shows a comparison of the Singular Value Spectrum between two points on the map QF shown in Figure 3. The curve in blue is a SV Spectrum on the maximum region of QF map and **h** = 300 μm/**dx** = 400 μm and the curve in red is a SV Spectrum on the minimum region of QF map and **h** = 100 μm/**dx** = 800 μm. This result illustrates an optimal combination of parameters in a blue curve, if we

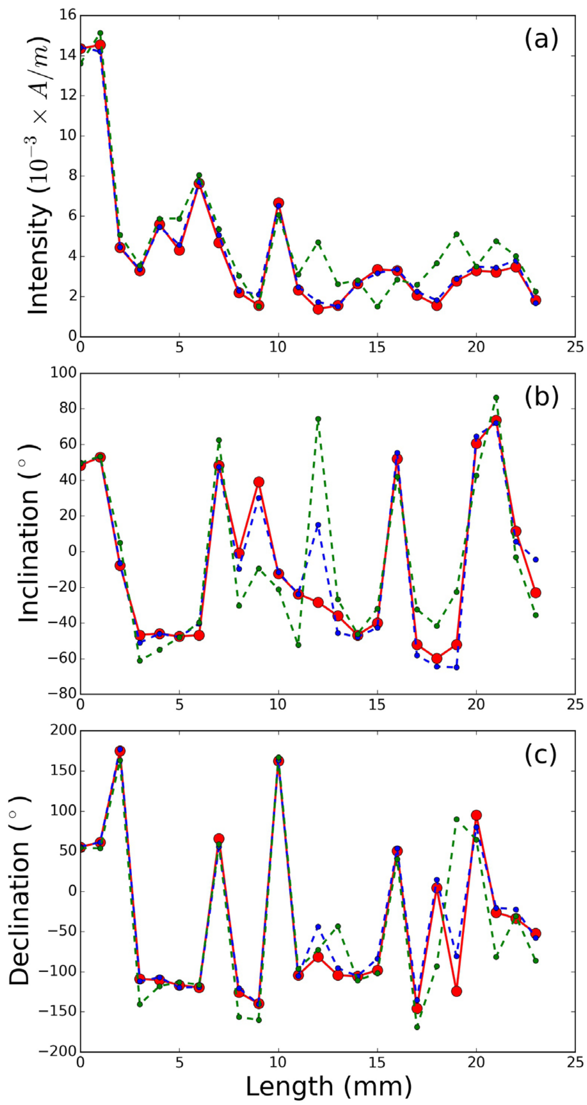

Figure 4. Comparison of the estimates after applying Reis *et al.* (2016) methodology using two points set on Figure 2. The red curve is the real magnetization distribution of simulation showed in Figure 1. The blue curve is the result using optimal acquisition design with **h** = 300 µm/**dx** = 400 µm. The green curve is the result of inversion using a low Quality Factor value with **h** = 100 µm/**dx** = 800 µm

compare with the curve in red. The result of application of the Reis *et al.* (2016) method (Fig. 4) shows the comparison among the real parameter (in red), the estimates using the highest QF value (in blue) and the lowest QF value (in green). The highest QF value produces best results for the estimates than the lowest QF value. It means that the SED technique can be a potential tool to construct a SMM experimental procedure.

4. Conclusion and discussion

We present some results showing how the linear Statistical Experimental Design technique can be used to set some acquisition parameters in the methodology proposed by Reis *et al*. (2016) for inverting SMM data using four observation planes around the sample. The preliminary results obtained with synthetic data show that the linear SED technique is a potential tool to set the sensor-to-sample distance and point density in the scanning stage for the purpose to improve the result on applying the method proposed by Reis *et al*. (2016). The construction of a QF map is a very important tool to analyze some possible acquisition schemes. We can set previously an optimal combination of sensor-to-sample distance and spatial resolution and how it impacts for estimating the magnetization distribution along a rock sample.

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