The role of poroviscosity in evaluating land subsidence due to groundwater extraction from sedimentary basin sequences

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RESUMEN
Este artículo describe la investigación que se lleva a cabo sobre el desarrollo del papel de la poroviscosidad en la determinación de subsidencia debido a la extracción de agua subterránea. La poroviscosidad incorpora tiempo cercano (llamado instantáneo), intermedio (llamado consolidación primaria) y tiempo lejano (creep o llamado compresión secundaria) en una teoría unificada de compresión esqueletal dependiente del tiempo. Requiere evaluar sólo una constante para la conducta unidimensional y en cualquier tiempo exhibe una relación semilogarítmica esfuerzo/tensión. La teoría de poroviscosidad debe proporcionar una exactitud mejorada de predicciones de subsidencia cuando secuencias sedimentarias saturadas de una cuenca son despresurizadas.

PALABRAS CLAVE: Poroviscosidad, subsidencia, extracción de agua subterránea, minería.

ABSTRACT
This paper describes research being carried out on the development of the role of poroviscosity in determining land subsidence due to groundwater extraction. Poroviscosity incorporates early-time (so-called instantaneous), intermediate (so-called primary consolidation) and late time (creep or so-called secondary compression) into one unified theory of time dependent skeletal compression. It requires only one constant to be evaluated for one-dimensional behaviour and at any time exhibits a semilogarithmic stress/strain relationship. Poroviscous theory should accordingly provide an improved accuracy of land subsidence prediction results when saturated sedimentary basin sequences are depressurized.

KEYWORDS: Poroviscosity, land subsidence, groundwater extraction, mining.

1. INTRODUCTION
The research concerns the development of land subsidence analysis, which uses poroviscosity as its constitutive relation, as opposed to the widely used poroelasticity theory, for modelling land subsidence due to groundwater extraction. The theory will be evaluated at selected sites in the Latrobe Valley of Victoria, Australia, where confined groundwater is extracted to maintain stability of large open cut lignite mines. Regional subsidence of up to almost 2 meters occurs around the mines. The main area of subsidence extends for up to 20 km from the Hazelwood mine where aquifer depressurization commenced in 1960. Because the subsidence is expressed as a large regional depression with no significant differential displacements at locations remote from the mines, the impact is restricted to gradient variation along water courses with the associated need to allow for land subsidence in flood estimates and in infrastructure design (Brumley, 1998).

Land subsidence occurs when groundwater or hydrocarbons are extracted from sedimentary basins. The phenomenon of subsidence due to fluid extraction from sedimentary basins is defined by Poland et al. (1972) as “sinking or settlement of the land surface, due to any of several processes. As commonly used, the term relates to the vertical downward movement of natural surfaces although small-scale horizontal components may be present. The term does not include landslides, which have large-scale horizontal displacements, or settlement of artificial fills”. With respect to aquifer depressurization, land subsidence may be considered as the manifestation at the land surface of the cumulative nonrecoverable compression component of a series of fast draining permeable zones and more importantly slow draining interbed lenses and confining layers between the aquifer systems, which extend down to bedrock in the sedimentary sequence.

Land subsidence analysis and the prediction of land subsidence due to groundwater and hydrocarbon extraction is a fairly new field of study. Fuller (1908) as cited in Helm (1982) was the first to suggest a link between fluid extraction and land subsidence. The next significant development in quantifying subsidence was the work of Terzaghi (1925), who
derived the one-dimensional consolidation equation. This permitted land subsidence to be quantified for the first time.

In the evolution of land subsidence theory Meinzer (1928) as cited in Poland (1984), recognized that water withdrawn from storage was released both by the compression of the aquifer and by the expansion of the water, and that reduction of storage, namely compression, may be permanent (inelastic) as well as recoverable (elastic).

Early observations of land subsidence caused by groundwater extraction were made by Rappley (1933) and Tibbetts (1933); as cited in Poland and Davis (1969), who were the first to identify land subsidence in the Santa Clara Valley, California, and Althouse (1935) as cited in Poland and Davis (1969), was the first to report land subsidence in the San Joaquin Valley, California. From the perspective of land subsidence the San Joaquin Valley is an important area. Since 1925 to the present the maximum global land subsidence value due to groundwater extraction has been recorded there. Subsequently, it has become the focus of ongoing land subsidence research. The land subsidence/groundwater extraction ratio in the San Joaquin Valley is similar to that in the Latrobe Valley (Evans, 1986).

The next milestone in the understanding of the manner, in which artesian aquifers release water from storage, was enunciated by Theis (1935) as cited in Poland (1984), who derived an analogy with the mathematical theory of heat conductivity. A very important deduction was made by Tolman and Poland (1940) as cited in Helm (1982), that the land subsidence in the Santa Clara Valley was not caused simply by declining artesian heads and the resulting compaction of permeable sands, but primarily by the non-recoverable compaction of slow draining clay layers within the confined system. The work of Jacob (1940) was important in the early development of understanding the response of elastic artesian aquifers to groundwater removal. Since the early 1920’s many land subsidence theories and models have appeared. Most of them have used the concept of elastic skeletal deformation to describe the flow of water from or into saturated sedimentary material. Thus poroelasticity has been the most common constitutive relation assumed directly or indirectly by groundwater hydrologists and geotechnical engineers (Helm, 1998). Elasticity theory has served as the basis for the solution of most practical soil mechanics and settlement problems since about 1925 (Poland, 1984). Many land subsidence theories and models have been formulated since Terzaghi (1925) first derived the one-dimensional consolidation theory and these owe something to Terzaghi’s pioneering theory. In the main these are variations of poroelasticity.

Land subsidence due to groundwater extraction is well recognized as a global phenomenon (Poland, 1984; Poland and Davis, 1969; Jelgersma, 1996; Nutralaya et al., 1996; Wells, 1996; Walker, 1992; Yamamoto, 1995; Toufigh and Sabet, 1995; Scott, 1979; Corapcioglu, 1983). The areal effects of land subsidence on a global scale have been detailed by Poland (1984) and Scott (1979).

Land subsidence can result in adverse effects on infrastructure caused by differential movement. Particularly at risk from the effects of land subsidence due to groundwater removal are low-lying areas adjacent to oceans, rivers and other bodies of water. During the 1970’s attention was focused on land subsidence because of increased public interest in the environmental impact of land subsidence (Schiller, 1975).

2. THEORIES OF LAND SUBSIDENCE

2.1. Current theory

Ever since the concept of elastic skeletal deformation was proposed implicitly by Meinzer (1923) and Terzaghi (1925) and explicitly by Jacob (1940) and Biot (1941); as cited in Helm (1998), poroelasticity has been used to describe the flow of water through saturated sedimentary material. The concept of poroelasticity has been the most common constitutive relation directly or indirectly assumed by groundwater hydrologists and geotechnical engineers. The concept requires that the soil skeletal frame deforms like an elastic solid. However, saturated sedimentary material does not actually deform in accordance with elastic analogy (Lambe and Whitman 1969). The use of poroelastic theory to describe the deformation of elastic and argillaceous sedimentary material makes certain mathematical assumptions which can lead to significantly inaccurate results.

In addition to the problems that result from assuming elastic deformation, poroelasticity also has the inherent disadvantage that it cannot account for secondary compression. Although postulated some six decades ago as an integral part of land subsidence (Merchant, 1939) as cited in Corapcioglu (1976), secondary compression has been relegated to the status of a qualitative component, if deemed necessary in land subsidence calculations. Corapcioglu (1983) states that various viscoelastic models have been proposed to address the deficiencies of the linear elastic theory and provide an extension of the classical elastic theory.

Until the theory of poroviscosity was postulated, secondary compression values could only be obtained from either the viscoelastic theory, or an extension of the poroelastic theory which used a time parameter (McNabb, 1960). The former of these two theories is more widely used than the latter. If poroelasticity is used, secondary compression values cannot be calculated and if required, are obtained by ad hoc means.
To put land subsidence prediction into perspective, prior to the work of Terzaghi (1925), methods for quantifying the time-dependence of soil deformation did not exist. The formulation by Terzaghi (1925) of the one-dimensional diffusion equation for fluid flow through deforming porous material, provided the basis on which all subsequent time dependent subsidence theories were based. Predicting land subsidence was not well understood into the 1950’s and considered too difficult to quantify and almost futile into the 1970’s. Gambolati and Freeze (1973) have made the valid point that most results up to 1969 were based on the classic one-dimensional theory. Poland (1969) stated that there was a need for more sophisticated models to be developed. Thus, this is a fairly new and evolving area of research.

2.2 Poroviscous theory

The poroviscous theory represents the behaviour of clastic and argillaceous material subjected to deformation caused by the flow of water through a saturated sedimentary material as a nonlinear viscous fluid. The grains and platelet packages that form the skeletal frame can be represented as particles of an idealized non-Newtonian fluid. Poroviscosity accomplishes three things. Firstly, it requires only one constant coefficient to be evaluated for one dimensional behaviour during a stress event. Thus, in this respect it is similar to poroelasticity, which also uses a constant to calculate subsidence during a stress event. Secondly, it incorporates instantaneous, primary and secondary compression into a unified theory of time dependent skeletal compression. Thirdly, at any time it exhibits a semilogarithmic compression relation. Subsidence calculations that use poroviscosity require only one calculation and need no further amendments or mathematical adjustments (Helm, 1998). This contrasts markedly with poroelasticity. The development of the poroviscosity theory as described in this paper should help to extend the boundaries of understanding of land subsidence, both in terms of calculation and prediction.

3. POROVISCOSITY CONSTITUTIVE RELATION

Section 3 and 4 briefly outline the constitutive relation and governing equation of the theory of poroviscosity (Helm, 1998) and are presented so that the development of the theory of poroviscosity in the ensuing sections can be understood. The nonlinear expression for the poroviscous constitutive relation is shown as Equation 1,

\[
\frac{\sigma'}{\sigma'} = \frac{\dot{e}}{\dot{\varepsilon}} + \frac{\ddot{e}}{A},
\]

where \(\sigma'\) is the effective stress, \(\varepsilon\) is strain and \(A\) is a poroviscous constitutive coefficient. One or two overlying dots indicate a first or second derivative respectively, with respect to time.

4. GOVERNING EQUATION FOR ONE-DIMENSIONAL CONSOLIDATION

The governing equation for one-dimensional consolidation is shown as Equation 2,

\[
du_x / dt + c \dot{\sigma}^2 u_x / \partial x^2 = R.
\]

where \(u_x\) is the cumulative displacement field of solids and \(c\) is the new poroviscous coefficient of consolidation which equals \(K\delta\sigma / Apg\); where \(K\) is the permeability, \(\rho\) the density of water, \(g\) is the acceleration due to gravity and \(\delta\sigma\) signifies the change in effective stress from its initial \(\sigma'_0\) value. The values of the traditional coefficient of consolidation \(c\), are very similar to the new poroviscous coefficient of consolidation \(c\), despite the different physical interpretation and evaluation process. The product of \(K\) and \(\delta\sigma\) and hence \(c\), can be treated as a constant (Helm, 1976). For a standard one-dimensional consolidation test, Helm (1987) has shown that \(R\) can be considered to be negligibly small for interior points within a soil specimen.

5. DEVELOPMENT OF POROVISCOSITY

In the derivation of the poroviscous constitutive relation Helm (1998) considered the case of a zero rate of change of stress, namely \(\dot{\sigma}' = 0\), or \(F(T) = 0\).

The development of the poroviscous theory which is the focus of this paper, encompasses the following:

(1) Derivation of a boundary condition for a constant rate of change of stress, namely \(\dot{\sigma}' = b\), or \(F(T) \neq 0\).

(2) Derivation of a solution for a displacement field of solids \(u_x\).

(3) Derivation of a dimensionless form of (2) and the production of unified consolidation \(U\)-time \(T\) plots.

(4) Embedment of: (a) a zero rate of change of stress \(F(T) = 0\), (b) a constant rate of change of stress \(F(T) \neq 0\), (c) the poroviscous constitutive relation, into COMPAC.

The derivation of the mathematics for the development of the poroviscous theory is shown in section 6. The poroviscous constitutive relation has been embedded into COMPAC, and results will be published when the work is validated. The embedment allows either the poroelasticity or poroviscosity constitutive relation to be used for land subsidence analyses. The hypothesis of the research is that the poroviscous theory will provide higher values of land subsidence than those obtained from poroelasticity.
Once validation is complete it is intended to use the land subsidence model COMPAC to analyze selected sites in the Tertiary sedimentary sequence of the Latrobe Valley, Victoria, Australia. The sequence comprises sands, silts, clays and thick coal seams with some interbedded basalt flows. Confined groundwater pressures have been lowered by up to 130 m.

COMPAC has been used to carry out land subsidence analyses in the USA at Pixley, California (Helm, 1975, 1976), San Jose, California, (Helm, 1978) and in Australia, the Latrobe Valley, Victoria, (Helm, 1984).

6. SOLUTION FOR ONE DIMENSIONAL CONSOLIDATION

The solution to Equation 2 for a constant rate of change of stress has been derived as follows:

For \( R = 0 \) and for the initial conditions

\[ u_x = 0 \quad 0 < x < H \quad t = t_o \]  

(2a)

and boundary conditions

\[ u_x = 0 \quad x = 0 \quad t > t_o \]  

(2b)

\[ \varepsilon = \varepsilon_n + A \ln \left\{ 1 + \frac{\dot{\varepsilon}}{A} \int_{t_o}^{t} \frac{\sigma' - \sigma'_0}{A} dt \right\} \quad x = H \quad t > t_o \]  

(2c)

where \( \sigma' = \sigma'_0 + f(t) \).

For \( n = 1 \), the solution to Equation 2 is:

\[
\begin{align*}
  u_x(x,t) &= H \sum_{m=1}^{M^2} \left\{ 1 - \exp\left[-M^2T \right] \right\} + A / \varepsilon_0 \ln \left\{ 1 + T/C + 1/C \int_{0}^{T} \frac{F(T)}{\exp\left[-M^2T \right]} \right\} \\
  &- A / \varepsilon_0 \int_{0}^{T} \left\{ 1 + F(T) \right\} \left\{ 1 + T/C + 1/C \int_{0}^{T} \frac{F(T)}{\exp\left[-M^2T \right]} \right\} \sin MX \\
\end{align*}
\]

(3)

where \( M = (2m-1)\pi/2, T = (t-t_o)/\tau, C = A / \dot{\varepsilon}_n, X = x/H, \tau = H^2/c, F(T) = 1/\sigma'_0f(t) \).

At \( x = H \), a dimensionless form of Equation 3 is:

\[ U = u_x(H,t)/\varepsilon_0H = U_1 + (A/\varepsilon_0)U_2 \]  

(4)

Boundary condition 2c corresponds to a constant rate of change of applied load at \( x = H \) and expresses a solution for constitutive relation 1. It is a general expression which enables \( n \) constant rates of change of stress \( \sigma'_k = b_n \), occurring over a time interval \( (t_k+1 - t_k) \), where \( 0 \leq k \leq n-1 \), to be incorporated in the consolidation equations.

If the general time-dependent boundary stress function \( f(t) \) shown in Equation 2d is represented by a series of ramp functions, as shown in Equation 5,

\[ f(t) = \sum_{k=0}^{n-1} b_k (t_{k+1} - t_k) + b_n (t-t_n) \]  

(5)

then if the strain for any specific single time interval is required, say \( t_o \), to \( t \), which has a constant rate of change of stress \( \sigma'_k = b_n \), Equation 6 can be used.

\[ \varepsilon = \varepsilon_n + A \ln \left\{ 1 + \frac{\dot{\varepsilon}_n(t-t_n)}{A + \dot{\varepsilon}_n b_n (t-t_n)^2 / 2A\sigma'_n} \right\} \]  

(6)

where \( t_o \) is the time of the last known stress value.

7. DISCUSSION

Figure 1 allows \( A/\varepsilon_0 = 1 \) and plots Equation 3 with primary consolidation \( U_1 \) and secondary consolidation \( U_2 \) having equal influence for the \( n \)th time interval which has a constant rate of change of boundary stress \( \sigma'_k = b_n \) for \( 0 \leq k \leq n \), where \( b_n \) is the slope of the \( k \)th constant rate of change of stress. Figure 1 represents typical poroviscosity type curves for \( F(T) = 0 \) shown dashed (Helm, 1998) and \( F(T) \neq 0 \) shown solid, where \( U \) the unified consolidation is plotted against the logarithmic scale of time \( T \).

Helm (1998) has shown that poroviscosity type curves for the case of constant stress (namely \( \sigma'_k = \sigma'_0 \)), closely match real laboratory consolidation data published by Taylor (1948). This implies a requirement for \( F(T) = 0 \) in Equation 3 and in Equation set 4. If \( \sigma'_k \) is set equal to zero in the Equation 2c then an exact match with the plots presented by Helm (1998) is obtained. Figure 1 shows good agreement for the curves \( F(T) = 0 \) and \( F(T) \neq 0 \). The indicated \( C \) values for the adjacent solid and dashed curves are identical. Hence, it is rea-
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reasonable to postulate that a similar close match for the $F(T)\neq 0$ poroviscosity type curves and real laboratory data, will also result from this development of the poroviscosity theory.

Helm (1998) has observed the simultaneity of primary and secondary compression. He has shown that $U_2$ dominates over $U_1$ after only a fraction of a second. This is an important observation since it implies that any attempted distinction between primary compression $U_1$ and secondary compression $U_2$ is blurred due to the timing of early measurements. The simultaneous behaviour of primary and secondary compression is not new. The earliest postulations of simultaneity were made by Merchant (1939), Taylor and Merchant (1940) and it has been observed by Tan (1957); as cited in Brutsaert and Corapcioglu (1976). Tan (1958, 1959) as cited in Brutsaert and Corapcioglu (1976), has modelled and researched secondary time effects. These postulations and observations of the synchronistic behaviour of $U_1$ and $U_2$ are significant and infer that instantaneous, primary and secondary compression all start at the same time. This aspect of the simultaneity of instantaneous, primary and secondary compression will have a significant impact on how the development of future subsidence theories will unfold. Previously the lack of research regarding secondary compression resulted from the belief that secondary compression was entirely different from primary compression and each had to be solved independently. This independent solution scheme was too complex and empiric approximations were used. With the introduction of poroviscosity they are no longer independent. Hence the solution process is simplified and the problem can be solved. However the relative magnitude of each compression component, namely instantaneous, primary and secondary, with respect to time has yet to be determined. It is hoped that this, along with the unified development of secondary compression will be incorporated in ongoing research into land subsidence predictions.

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