



## NEURO FUZZY LOGIC INFERENCE OF S-RATIOS, THROUGHOUT A STRATIGRAPHIC SECTION FROM THE LLANOS FORELAND BASIN (COLOMBIA), USING ROCK MAGNETIC AND PETROPHYSICAL DATA

Vincenzo Costanzo-Álvarez<sup>1</sup>, Victoria Camacho<sup>1</sup>, Milagrosa Aldana<sup>1</sup>,  
Diego López-Rodríguez<sup>2(\*)</sup>, Germán Bayona<sup>3</sup>

<sup>1</sup> *Departamento de Ciencias de la Tierra, Universidad Simón Bolívar, Caracas, Venezuela. E-mail: vcosta@usb.ve, maldana@usb.ve*

<sup>2</sup> *Laboratorio de Física Teórica de Sólidos, Escuela de Física, Universidad Central de Venezuela, Caracas, Venezuela.*

<sup>3</sup> *Corporación Geológica Ares, Bogotá, Colombia. E-mail: gbayona@cgares.org*

### Abstract

The Neuro Fuzzy Logic hybrid algorithm is used here to infer S-ratio, via experimental volume magnetic susceptibility ( $\kappa$ ), SIRM/ $\chi$  and (gamma ray) shale volume ( $V_{sh}$ ) data, in a 512 meters sedimentary section (100 samples) of the stratigraphic well Saltarin 1A (Llanos foreland basin, Colombia). This section includes the Guayabo and part of the León formation. Computational tests using 62 and 100% of the data available show that the best inference for the whole well is obtained when a Gaussian membership function is employed with a semilog relationship for the 62 samples ( $\log\kappa$ , SIRM/ $\chi$ ,  $V_{sh}$  with 4 fuzzy rules [2,2,1]) and a direct relationship for the 100 samples ( $\kappa$ , SIRM/ $\chi$ ,  $V_{sh}$  with 6 fuzzy rules [2,3,1]). The correlation parameters are RMSE = 0.1147,  $R^2 = 0.6778$  and RMSE = 0.1626,  $R^2 = 0.54$  in each case. These results seem to indicate that, in complex geological settings, S-ratio is related not only to the redox conditions of the sedimentary paleoenvironments (i.e.  $Fe_3O_4/Fe_2O_3$  ratio), but also to the lithological contrasts that are accompanied by variations of distinct types of magnetic minerals (e.g. Fe sulphides) and magnetic grain size distributions.

### Resumen

Un algoritmo híbrido de redes neuronales difusas es usado en este estudio para inferir los valores del cociente S a partir de datos experimentales de susceptibilidad magnética volumétrica ( $\kappa$ ), SIRM/ $\chi$  y volumen de arcilla ( $V_{sh}$ ) calculado a partir de un registro gamma ray. El estudio fue realizado en una sección sedimentaria de 512 metros (100 muestras) proveniente del pozo estratigráfico Saltarín 1A (cuenca de los Llanos orientales de Colombia). Esta secuencia incluye las formaciones Guayabo y León. Numerosas pruebas computacionales, usando el 62 y el 100% de los datos disponibles, permiten concluir que la mejor inferencia es aquella que se obtiene con una función Gaussiana de membresía y una relación semilog aplicada a 62 muestras ( $\log\kappa$ , SIRM/ $\chi$ ,  $V_{sh}$  con 4 reglas difusas [2,2,1]) o una relación directa aplicada a las 100 muestras ( $\kappa$ , SIRM/ $\chi$ ,  $V_{sh}$  con 6 reglas difusas [2,3,1]). Los parámetros de correlación serían RMSE = 0.1147,  $R^2 = 0.6778$  y RMSE = 0.1626,  $R^2 = 0.54$  en cada caso. Estos resultados parecen indicar que, en situaciones geológicas complejas, el cociente S está relacionado no sólo con las condiciones óxido reductoras del paleoambiente sedimentario (i.e.  $Fe_3O_4/Fe_2O_3$  ratio) sino también con los contrastes litológicos que vienen acompañados por variaciones en los distintos tipos de minerales magnéticos presentes (e.g. sulfuros de Fe) y en las distribuciones de tamaños de granos magnéticos.

---

(\*) *Present address: Facultad de Ciencias Físicas, Universidad Complutense de Madrid, Ciudad Universitaria, 28040 – Madrid, España*



## Introduction

Geophysical and geological problems commonly entail systems with a large number of parameters interacting between each other in a complex way. Such interactions are mostly non-linear and non-random resulting in an increasing scatter of experimental data points that blurs up any likely associative trend among them. In nearly all cases, the difficulty of finding a relationship connecting naturally continuous variables is tackled by dividing them into a number of arbitrary groups. Thus, some local deterministic, empirical or semi-empirical models are obtained, namely a linear or multilinear formula that correlates experimental variables for all of these groups of data (Finol et al., 2001a). Conversely, the Neuro Fuzzy logic, a hybrid algorithm that combines fuzzy logic with neural networks, describes these variables in a more natural and rigorous way. Based on an automatic pattern recognition technique, the fuzzy logic method searches for the different sets of data involved in a complex system and for the empirical relationships between them.

In this preliminary work we have used the Neuro Fuzzy logic technique to infer S-ratio from rock magnetic experimental data, namely magnetic susceptibility ( $\kappa$ ) and Saturation Isothermal Remanent Magnetization normalized by the specific susceptibility ( $SIRM/\chi$ ), as well as by petrophysical data, namely shale volume ( $V_{sh}$ ) calculated from a gamma ray log that covers the first 512 meters of the Colombian stratigraphic well Saltaín 1A (Colombian Llanos foreland basin, figure 1a).

The S-ratio, a rock magnetic index that accounts for the relative contributions of low and high coercivity material to the total saturation isothermal remanent magnetization in a sample, has been determined here according to the definition by Bloemendal et al. (1992). However, by obtaining empirical relationships that correlate S-ratio with magnetic susceptibility,  $SIRM/\chi$  and  $V_{sh}$  we are actually exploring how this magnetic parameter is tied up to the concentration of magnetic minerals, their grain size distributions and the percent of shale in the different strata analyzed. Our ultimate goal is to apply the Neuro Fuzzy logic technique as a unbiased quantitative tool for pattern recognition of the major stratigraphic units involved. The Saltaín 1A seems to be an ideal natural scenario for such a purpose since it shows numerous lithological contrasts that give rise to a complex geological system (Bayona et al., 2008).

## Stratigraphic setting

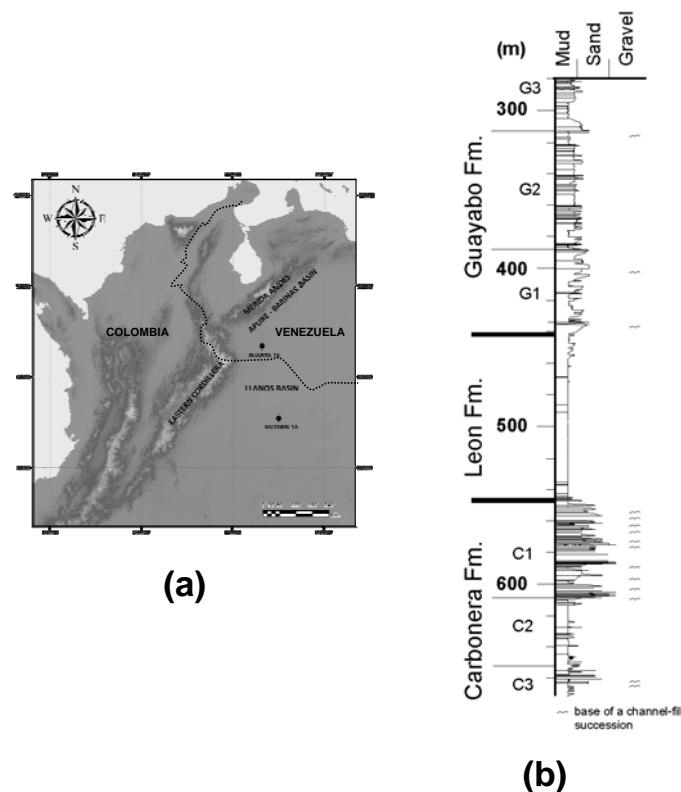
Figure 1b shows a sketch of the stratigraphic column of Saltaín 1A. The deepest formation studied in this work is León (441.8 to beyond 512 m), a muddy sequence of sediments from a fresh-water lacustrine system. On top of León lies the Guayabo formation that was divided in 6 lithological units (Bayona et al., 2008):

G1 (388 to 441.8 m) and G2 (312.9 to 388 m), the two lower units, consist of green-colored laminated mudstones grading to sandstones interbedded with light-colored massive mudstones with ferruginous nodules. These lithologies were interpreted as the sedimentation from a fluvio-deltaic system changing to more continental sediment accumulation in fluvial floodplains.

G3 (271.5 to 312.9 m) and G4 (205.5 to 271.5 m), the overlying units, are dominantly mudstones and siltstones that accumulated in fluvial flood plains. The unit G3 has more evidence of subaerial exposure (light-colored mudstones, formation of ferruginous nodules), whereas preservation of coal beds and laminated mudstones in unit G4 indicates less subaerial exposure of the flood plains.

G5 (81.6 to 205.5 m) consists of feldspar-rich sandstones that record the filling of fluvial channels.

G6 (0 to 81.6 m), the uppermost unit of the Guayabo formation, records a change to floodplain with evidence of subaerial exposure.



**Figure 1 :** Geographical setting and stratigraphic column of the well Saltarín 1A

## Methods

For the characterization of the different lithostratigraphic units, through the inference of S-ratio, we used a hybrid Adaptive Neuro Fuzzy Inference System (ANFIS) with five layers that can be interpreted as a neural network with fuzzy parameters or a fuzzy system with distributed parameters. This hybrid Neuro Fuzzy system is equivalent, under some constraints, to a Takagi, Sugeno, Kang (TSK) model (Finol and Jing, 2002). To train our fuzzy model we used S-ratio ( $IRM_{-0.03T}/SIRM_{+3T}$ ), as output and  $\kappa$ ,  $SIRM/\chi$  and  $V_{sh}$  as input variables. The 100 samples analyzed were taken at about every 5 meters of depth down to approximately 512 meters. They are mudstones, sandy and silty mudstones and sandstones from the Guayabo and León formations.

We introduced the  $\kappa$  training data in either semilogarithmic or direct form (i.e.  $\log \kappa$  or  $\kappa$  respectively) and tried out with different sets of one ( $\kappa$  or  $\log \kappa$ ), two ( $\kappa$  or  $\log \kappa$  and either  $SIRM/\chi$  or  $V_{sh}$ ) or three ( $\kappa$  or  $\log \kappa$ ,  $SIRM/\chi$  and  $V_{sh}$ ) independent input variables. Tests were also carried out using different combinations and numbers of fuzzy rules (up to 6). Membership functions employed in all the trials were either linear, triangular, bell, pi, or Gaussian. The first tryouts were done with only 62 samples randomly chosen and evenly distributed along the whole well. Afterwards, these results were checked using all the 100 samples. In each case inferred S-ratio values were compared with their experimental counterparts. To quantify the performance of the inference, we applied the  $R^2$  between inferred and experimental S-ratio data, and the Root Mean-Square Error ( $RMSE$ ) values. Figures 2a, b, c and d presents the profiles of volume magnetic susceptibility ( $\kappa$ ), S-ratio,  $SIRM/\chi$  and  $V_{sh}$  plotted against stratigraphic level.

## Computational Results

Figure 3 shows a cross plot for experimental S-ratio and  $\kappa$  data that includes both, 62 samples out of 100 analyzed (in red) and all the 100 samples analyzed (in blue). Initially we tried to infer S-ratio values using a simple linear regression in order to describe the behavior observed in this cross plot, as well as those for S-ratio versus  $SIRM/\chi$  or  $V_{sh}$ . None of these fittings provides a satisfactory result



due to the high scattering of experimental data points. The best linear fit might be the semilog ( $\kappa$ ) relationship shown in figure 3. This curve resembles the non-linear mathematical form of the function that relates the value of S-ratio to the magnetite weight percentage for an array of artificial samples systematically mixed from magnetite and hematite (Heslop, 2009; Frank and Nowaczyk, 2008). The low  $R^2$  values obtained in each case confirm the poor performance of the linear regression approach and suggest that non-linear methods could be more adequate to describe the relationship between these data. Hence we used the alternative Neuro Fuzzy logic method to find a family of local fitting functions for different sub groups of experimental data points (Finol et al., 2001b).

After numerous trials with the ANFIS, the best inference was accomplished with a Gaussian membership function and a semilog relationship for the 62 samples (i.e.  $\log\kappa$ ,  $SIRM/\chi$  and  $V_{sh}$ ) or a direct relationship for the 100 samples (i.e.  $\kappa$ ,  $SIRM/\chi$  and  $V_{sh}$ ). These results are shown in table 1 and figure 4.

In figures 4b and d the inferred data (blue) is confronted against the experimental data (red) showing, in both cases, a good agreement with  $RMSE = 0.1147$  and  $R^2 = 0.6778$  for the linear relationship between inferred and experimental data shown in figure 4a, and  $RMSE = 0.1626$  and  $R^2 = 0.54$  for the linear relationship between inferred and experimental data shown in figure 4c.

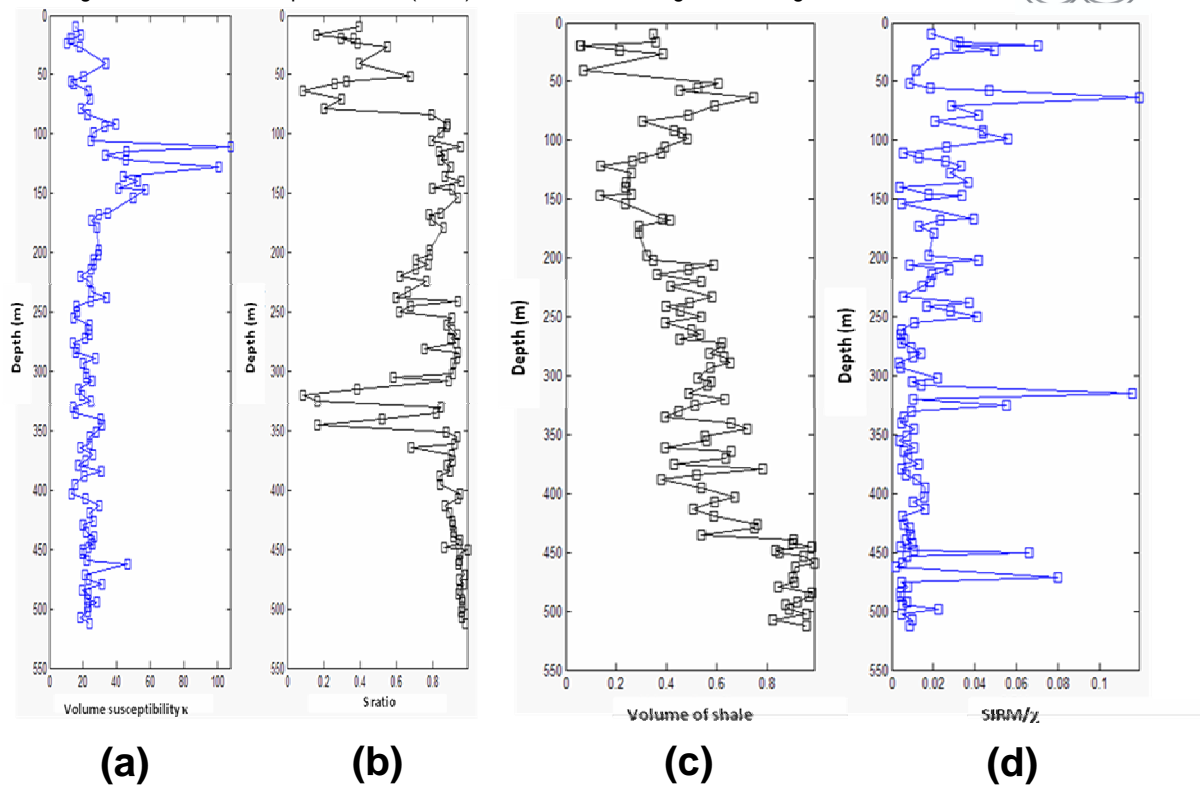
## Discussion

Figure 3 illustrates the difficulties involved in finding a linear predictive model out of experimental S-ratio and  $\kappa$  data for a set of heterogeneous natural rock samples. The same could be said about the possible relationships between S-ratio and either  $SIRM/\chi$  or  $V_{sh}$  values. We argue that the scattering of these data might be the consequence of dealing with natural samples that have, for each depth level, variable amounts of magnetic minerals other than magnetite and hematite (i.e. Fe-sulfides), distinct distributions of magnetic mineral grain sizes and different contributions of paramagnetic fractions.

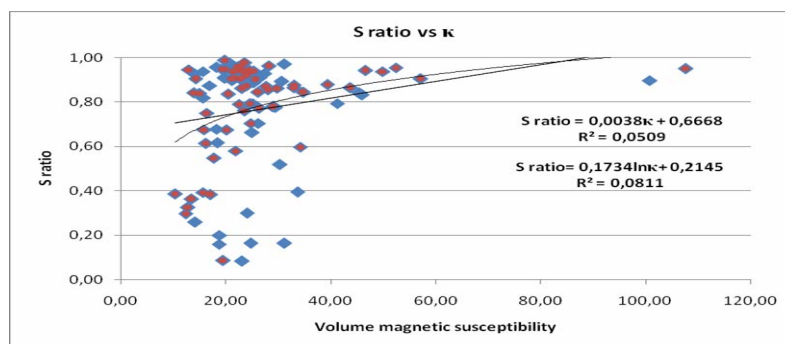
These results advocate not only for the use of an alternative non linear method such as the Neuro fuzzy logic hybrid algorithm, but also for the choice of an optimal number of parameters (e.g.  $\kappa$ ,  $SIRM/\chi$  and  $V_{sh}$ ) that would take into account most of the mineralogical complexities and variations of the different strata analyzed. For instance, it is widely accepted that, in magnetite-rich sediments, volume magnetic susceptibility ( $\kappa$ ) is a sensible index that measures the concentration of this mineral, however  $\kappa$  does not seem to be as sensitive as  $SIRM/\chi$  to changes in magnetic grain-sizes. Conversely, unlike to magnetite, the susceptibility of pyrrhotite depends on the internal field only for grains smaller than  $30 \mu\text{m}$ , whereas coercivity values increase with decreasing grain sizes (Sagnotti, 2007).  $\kappa$  also integrates contributions from paramagnetic as well as hard and soft ferromagnetic fractions, whereas S – ratios and  $SIRM/\chi$  differentiate contributions of hard and soft fractions only.

Lithological contrasts along the Saltarín 1A stratigraphic well are reflected by both, variations of  $V_{sh}$  and magnetic properties of its different strata. As a matter of fact, Da Silva et al. (2010) have pointed out that G5 unit (upper Guayabo formation), dominated by magnetite, is completely different from lower Guayabo and León formations due to the increasing presence of magnetic phases other than magnetite and hematite. Lower Guayabo (figure 1b) corresponds to a transition zone that goes, downwards, from alluvial plains sediments accumulated in a reducing environment, and the oxidized paleosols of units G1 and G2 of the Guayabo formation, to the lacustrine settings that characterize the León and Carbonera formations (Bayona et al, 2008).

Our best results (Table 1, figure 4), reveal the importance of training the Neuro Fuzzy net with all the four parameters we are considering (i.e. S-ratio,  $\kappa$ ,  $SIRM/\chi$  and  $V_{sh}$ ) in order to accommodate for the drastic lithological and rock magnetic variations between the bottom of Guayabo and the top of León. Indeed, figure 4 depicts, along the whole well, a good agreement between measured and inferred S-ratio values. Conversely, in some of the preceding tests that used less than four of these parameters to train the system, it is evident the inability of the system for finding a unique set of fuzzy rules that could properly infer S-ratio over every stratigraphic unit involved.



**Figure 2:** Profiles of (a) volume susceptibility ( $\kappa$ ), (b) S-ratios, (c) Shale volume (calculated from gamma ray data) and (d)  $SIRM/\chi$  values along the first 512 meters of the stratigraphic well Saltarín 1A.



**Figure 3:** Cross plot of experimental S-ratio and  $\kappa$  values, including 62 (red) and 100 (blue) data points measured in Saltarín 1A. Empirical formulae using a simple linear regression were also obtained for a direct and semilog relationship between these two parameters.

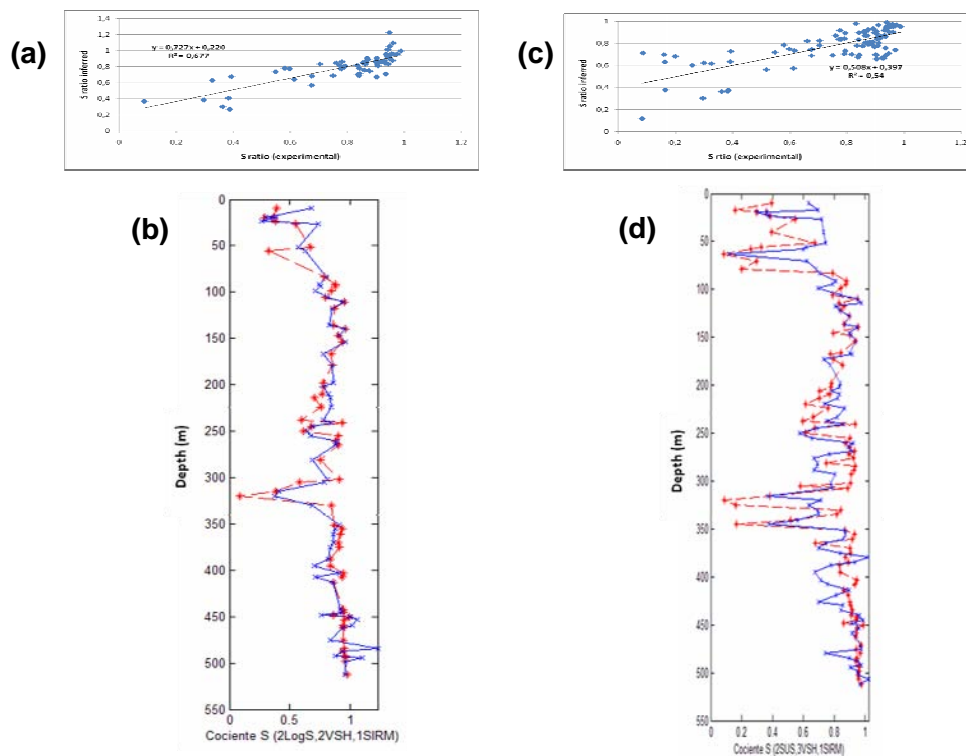
Thus we can conclude that, in complex geological settings like the stratigraphic section of Saltarín 1A, S-ratio seems to be related not only with the redox conditions of the sedimentary paleoenvironments (i.e.  $Fe_3O_4/Fe_2O_3$  ratio), but also with lithological contrasts that are accompanied by variations of different types of magnetic minerals (e.g. Fe sulphides) and magnetic grain size distributions.

**Acknowledgments:** Samples were generously provided by Alejandro Mora (Hocol, Bogotá). He also made available important unpublished geological and geophysical information. This research was partially funded by the Decanato de Investigación y Desarrollo and Dirección de Desarrollo Profesional, both at the Universidad Simón Bolívar (Caracas, Venezuela), via research grants and FONACIT LOCTI grants to M. D. and M. A.



**Table 1 Summary of the best results obtained applying ANFIS to either 62 samples chosen randomly or the whole collection of 100 samples**

Parameters	RMSE	R <sup>2</sup>	Parameters of the membership functions [Δh <sub>1/2</sub> center]	Fuzzy Rules
logκ, V <sub>sh</sub> , SIRM/χ <sub>i</sub> ; 4 Rules (2,2,1)	0,1147	0,6778	logκ:[0.413 0.9803] logκ:[0.4232 2.01] V <sub>sh</sub> :[0.1982 0.004689] V <sub>sh</sub> :[0.1301 1.039] SIRM/χ <sub>i</sub> :[0.1158]	CS = 1.577 logκ + 0.527 V <sub>sh</sub> - 3.525 SIRM/χ <sub>i</sub> - 1.263 CS = - 28.19 logκ + 9.47 V <sub>sh</sub> - 9.301 SIRM/χ <sub>i</sub> + 22.97 CS = 0.6505 logκ - 0.747 V <sub>sh</sub> - 3.757 SIRM/χ <sub>i</sub> - 11.81 CS = - 33.64 logκ - 14.1 V <sub>sh</sub> + 49.47 SIRM/χ <sub>i</sub> + 75.5
κ, V <sub>sh</sub> , SIRM/χ <sub>i</sub> ; 6 Rules (2,3,1)	0,1626	0,54	κ:[41.34 10.34] κ:[41.34 107.7] V <sub>sh</sub> :[0.1901 0.2697] V <sub>sh</sub> :[0.3125 0.4647] V <sub>sh</sub> :[0.13 0.9061] SIRM/χ <sub>i</sub> :[0.1192]	CS = -0.02696 κ + 2.818 V <sub>sh</sub> + 7.359 SIRM/χ <sub>i</sub> - 1.063 CS = 0.08823 κ - 1.403 V <sub>sh</sub> - 6.017 SIRM/χ <sub>i</sub> + 1.148 CS = -0.2019 κ - 8.975 V <sub>sh</sub> + 13.26 SIRM/χ <sub>i</sub> + 8.57 CS = -0.05042 κ + 15.13 V <sub>sh</sub> + 40.92 SIRM/χ <sub>i</sub> - 0.52 CS = 0.09173 κ - 25.1 V <sub>sh</sub> - 67.38 SIRM/χ <sub>i</sub> + 1.46 CS = - 0.8885 κ + 62.31 V <sub>sh</sub> - 75.57 SIRM/χ <sub>i</sub> + 10.06



**Figure 4:** S-ratio inference using ANFIS. In a and b the network was trained with 62 data points of S-ratio, log κ, V<sub>sh</sub> and SIRM/χ<sub>i</sub> (4 fuzzy rules [2,2,1]). In c and d we applied 6 fuzzy rules [2,3,1] to the whole collection of 100 data points. These rules were obtained by training the system with 62 data points of S-ratio, κ, V<sub>sh</sub> and SIRM/χ<sub>i</sub>. In a and d blue lines stand for the inferred S-ratio data whereas the red ones represent the experimental data. Cross plots in a and c show the linear relationship between these two sets of S-ratio data.

**References**

Bayona, G., Valencia, A., Mora, A., Rueda, M., Ortiz, J. and Montenegro, O., 2008, Estratigrafía y procedencia de las rocas del Mioceno en la parte distal de la cuenca antepais de los Llanos de Colombia: Geología Colombiana, Vol. 33, 23 - 46.



Bloemendal, J., King, J.W., Hall, F.R. and Doh, S.J., 1992, Rock magnetism of Late neogene and Pleistocene deep-sea sediments: relationship to sediment source, diagenetic processes and sediment lithology. *Journal of Geophysical Research*, 97, 4361-4375.

Da Silva A., Costanzo-Álvarez V., Hurtado N., Aldana M., Bayona G., Guzmán O., López-Rodríguez D. 2010, Possible correlation between miocene global climatic changes and magnetic proxies, using neuro fuzzy logic analysis in a stratigraphic well at the Llanos foreland basin, Colombia *Studia Geophysica et Geodaetica*, 54, 607 - 631

Finol, J., Guo, Y. and Jing, X., 2001a, Fuzzy Partioning Systems for Electrofacies Classification: a case study from the Maracaibo Basin: *Journal of Petroleum Geology*, 24(4) 441-548.

Finol, J., Guo, Y. and Jing, X. 2001b, A rule based fuzzy model for the prediction of petrophysical rock parameters. *Journal of Petroleum Science and Engineering*, 29 97-113.

Finol, J. and Jing X., 2002, Permeability prediction in shaly formations: The fuzzy modeling approach. *Geophysics*, 67(3), 817-829.

Frank, U. and Nowaczyk, N.R., 2008, Mineral magnetic properties of artificial samples systematically mixed from haematite and magnetite. *Geophysical Journal International*, 175, 449-461.

Heslop, D. , 2009, On the statistical analysis of the rock magnetic S-ratio. *Geophysical Journal International* 178, 159-161

Sagnotti, L., 2007, Iron Sulfides In: *Encyclopedia of Geomagnetism and Paleomagnetism*, David Gubbins and Emilio Herrera-Bervera editors, Springer , 454-459.